Available online at www.sciencedirect.com
SCIENCE DIRECT®

International Journal of Heat and Mass Transfer 48 (2005) 1494–1504

International Journal of HEAT and MASS TRANSFER

www.elsevier.com/locate/ijhmt

Effect of oscillatory motion on heat transfer at vertical flat surfaces

H. Gomaa *, A.M. Al Taweel

Department of Chemical Engineering, Dalhousie University, P.O. Box 1000, Halifax, NS, Canada B3J 2X4

Received 29 March 2004; received in revised form 18 October 2004
Available online 15 December 2004

Abstract

The effect of oscillations on heat transfer at vertical surfaces is investigated and a model is developed that predicted both the transient and time average heat transfer rates. The transient behavior of the heat transfer indicates the presence of an oscillatory component superimposed on a larger steady one that does not reach zero during flow reversal. This was explained in terms of the interaction between a "quasi-steady oscillatory" mechanism near the leading edge, and a "pseudo-steady diffusive" far from it. The analysis further revealed that the time average heat transfer rate can be adequately estimated using a mixed "forced-natural" convections correlation, with the forced convection component estimated based on the time average oscillatory Reynolds number $Re_v = awL/\nu$. The agreement between the model predictions and the experimental measurements makes it applicable for predicting heat transfer characteristics and velocity fluctuations near heated vertical surfaces in presence of oscillatory motion. The model is also applicable for predicting heat transfer rates under conditions where oscillatory motion is used to achieve specificity in temperature control without affecting process residence time, such as in biomedical and biochemical applications. The modest heat transfer enhancement (<2) due to oscillatory motion is attributed to the small convective term in the energy equation, which is consistent with previous investigations where increasing the axial temperature gradient in presence of oscillatory motion was shown to achieve much higher heat transfer enhancement.

Keywords: Heat transfer; Oscillatory motion; Enhancement; Process intensification

1. Introduction

Oscillatory flows has been known to result in higher rates of heat and mass transfer, and numerous studies have been done to understand its characteristics in different systems such as reciprocating engines, pulse combustors, and chemical reactors to cite a few [1–3]. The work of Kersweg [4–6] is of particular interest since it demonstrated the possibility of achieving significant heat transfer enhancement by increasing the axial temperature gradient in presence of oscillatory motion. This has led to the revised interest in the subject matter with the advent of new technological applications such as high-performance Stirling engines and pulse tube cryocoolers where great deal of effort is being devoted towards their development for military, space and

* Corresponding author. Address: Department of Chemical Engineering, Dalhousie University, P.O. Box 2103, Port Elgin, ON, Canada N0H 2C0. Fax: +1 519 797 1956.
E-mail addresses: hgomaa@dal.ca, gomaah@bmts.com (H. Gomaa).

0017-9310/$ - see front matter
doi:10.1016/j.ijheatmasstransfer.2004.10.017

Nomenclature

a	amplitude of plate vibration [mm]
f	frequency of electrode vibration [Hz]
g	acceleration of gravity [mm/s^2]
Gr	Grashof number ($Gr = \lambda g L^3/\nu^2$)
h	heat transfer coefficient [mm/s]
L	length of surface active area [mm]
p	dimensionless function, Eq. (10)
Nu	Nusselt number ($Nu = hL/\alpha$)
Pr	Prantdl number ($Pr = \nu/\alpha$)
Re	Reynolds number ($Re = uL/\nu$)
S	shear rate at the solid–liquid interface [s^{-1}]
t	time [s]
T	temperature [^{0}K]
u	x-directional velocity [mm/s]
v	y-directional velocity [mm/s]
x	positions along the flat plate [mm]
y	positions normal to the flat plate [mm]
$\breve{y}$	centre of mass, Eq. (37) [mm]

Greek symbols

α	thermal diffusion coefficient [mm^2/s]
β, γ	coefficients, Eq. (18)
λ	coefficient of volumetric expansion, Eq. (8)
ν	kinematic viscosity [mm^2/s]
ρ	fluid density [g/mm^3]
ω	circular frequency of vibration, $2\pi f$ [s^{-1}]
ϕ	dimensionless temperature [$\phi = (T - T_\infty)/(T_w - T_\infty)$]
η	dimensionless variable, Eq. (9)
ψ	stream function, Eq. (10)
ξ	similarity variable, Eq. (26)
Γ	Gamma function

Subscripts

o	conditions outside the boundary layer
f	forced convection
n	natural convection
v	time average vibrational value
w	conditions at the surface
∞	conditions in the bulk of the fluid

civilian applications [7,8]. The subject has also received increased attention in the chemical industry with the emergence of applications requiring specificity in mixing and temperature control that could not be achieved by conventional mixing or by increasing flow rates. For example, processes which require long residence time but nevertheless require careful temperature control, can benefit greatly from using oscillatory motion, since low bulk flow velocity permits smaller mixer, while the heat transfer rate can be controlled independently through the oscillatory conditions [9]. Similarly, oscillatory motion can be applied for designing compact high-performance heat exchangers with high precision temperature control [10–12].

Oscillatory motion can be produced by either pulsating the fluid [13,14], or vibrating the solid surface [15,16], which vary according to system configuration. Although both approaches achieve the same objective of creating an oscillatory velocity vector between the solid surface and the fluid medium, the former is more energy efficient, since the power consumption is focused on the main transfer resistance at the solid–liquid interface instead of being dissipated into the bulk of the liquid [17]. Depending on the ratio of the oscillation amplitude to the mean flow velocity, oscillatory motion can result in conditions without flow reversal at low amplitude to velocity ratio, or to conditions where flow reversal occurs periodically as the ratio increases. Based on published information, significant heat transfer enhancement occurs under conditions when the oscillating amplitude is sufficiently large to cause flow reversal [12,8].

In order to achieve the full potential of oscillatory motion for process intensification and for designing high-performance heat transfer devices, proper understanding of the enhancement mechanisms and the fundamentals of the thermal and fluid mechanics associated with oscillatory flows is necessary. Review of the literature revealed that there is still deficiency in this area, and the question of understanding the mechanisms involved and their contribution to the enhancement factor still poses an issue of fundamental importance from the standpoint of both basic research, and the development of effective scale up methodologies. For example, and as pointed out by Drummond and Lyman [18], and Al Taweel and Landau [19], enhancement due to the formation of turbulent eddies is significantly different from that caused by acoustic streaming, or boundary layer thinning, and the parameters affecting each are also different. In another example, while Ralph [20], and Mackley [21] attributed performance improvement under oscillatory flows to standing vortex wave formation, both Rodgers and Sparks [22], and Abel [23] reported that much of it, if not most, is due to the negative pressure portion generated each half of the oscillatory cycle. Both mechanisms, on the other hand are different from enhancement due to self-sustained resonant transport reported by Nishimura et al. [24].

The fact that there are more than one heat transfer mechanism involved in most engineering applications where oscillatory motion is used, makes it necessary to take into consideration the interaction between them for predicting the overall enhancement factor. The investigation done by Dec et al. [12] for heat transfer in a pulse combustor tail pipe is an example of such interaction, where it was shown that the observed heat transfer enhancement could not be explained using one single mechanism, and that the combined effect of different mechanisms involved must be taken into account for proper design and scaleup purposes. Another example, is the case of using oscillatory motion for enhancing processes controlled by natural convection. Under such conditions, the enhancement factor becomes dependent on the combined effect of both natural convection and the oscillatory motion. Examples of such applications include the dream pipes, pulsed heat pipes, and oscillation-controlled heat transport tubes, where knowledge of both axial and lateral heat transport to and from the wall is needed for determining their heat transfer characteristics [25–27]. It also includes oscillatory electrochemical reactors, particularly if temperature gradient exists between electrodes surfaces and the adjacent fluid [28], cooling of micro-electronics equipment, and measurements of pulsation characteristics in a boundary layer along heated vertical surface using hot film anemometer [29–32].

In spite of its significance, few studies have been conducted on the combined effect of natural convection and oscillatory motion on heat transfer characteristic, and most of them included simplified assumptions and approximations, such as ignoring the effect of flow reversal. In this paper, the effect of bouncy force and oscillatory motion on both fluid mechanics and heat transfer at vertical surfaces is analyzed. The heat transfer characteristics are discussed with reference to the major physical mechanisms at work, which stems from the interaction between the velocity and temperature oscillations, and the mixed convection phenomena induced by the buoyancy effect is discussed. The model predictions are compared with experimental data from previous investigations.

2. Theoretical analysis

2.1. Background

The subject of diffusion across unsteady boundary layer has been addressed by several investigators where it was demonstrated that under conditions of slow shear variation (low frequencies), a quasi-steady assumption can be made and Leveque [33] one-third-power law in which the flux is proportional to the one-third power of the local shear rate, can be used. In cases, where the variation of the shear rate is fast (high frequencies), significant deviation from the quasi-steady state predictions occurs, and knowledge of the dynamic behavior of the thermal boundary layer is required. This was analyzed for small amplitude oscillations, where the boundary-layer equation can be linearized and solved asymptotically or numerically [34–37]. For large amplitude oscillations where shear reversal occurs, the basic simplification of the boundary-layer theory is lost since the "leading edge" of the body changes ends, and the velocity profile at a given point along the surface becomes not only dependent on the upstream, but also on the downstream flow conditions at different times. For such conditions, Pedley [38] developed a model based on combining an asymptotic quasi-steady expansion solution with a purely diffusive solution during shear reversal that fairly predicted experimental results. The basis of the analysis was also confirmed by Watkins and Herron [39] who showed that for a plate oscillating with a zero mean velocity, a Stokes layer will exist over the plate surface except for a small distance from the plate leading edges. The problem was also analyzed by Kaiping [40], Steenhoven and Van Beucken [41], and Mao and Hanratty [42,43], who examined numerically the unsteady forced convective transfer under reversing and non-reversing shear flow conditions, and reported similar results.

In presence of natural convection, the situation is more complex due to the coupling of the momentum and energy equations, and most investigations resulted in experimentally developed correlations for the time average transfer component [15,44–46]. Analyses of the dynamic behavior were limited to either cases of small amplitude oscillation with no flow reversal [47–49], bodies of revolution, where acoustic streaming is the basic mechanism at work [50,51], or to cases of stationary surfaces in an oscillatory temperature or gravity fields [52–55].

2.2. Model development

Consider the vertical thin flat plate in Fig. 1 heated to a uniform temperature T_w and placed in an ambient temperature T_∞ where a fully developed steady state natural convection is established due to density difference. Assume that the property variation with temperature is limited to density and its effects on the buoyancy term in the momentum equation (Boussinesq approximation). As the plate oscillates harmonically in its own plane, and away from its leading edge, a much thinner layer (Stokes layer) carried by the plate will develop, and diffusion across such layer will contribute to the DC heat transfer component. Near the plate leading edge, on the other hand, a quasi-steady state forced convection mechanism is dominant and extends from the plate leading edge to a point on the surface when a

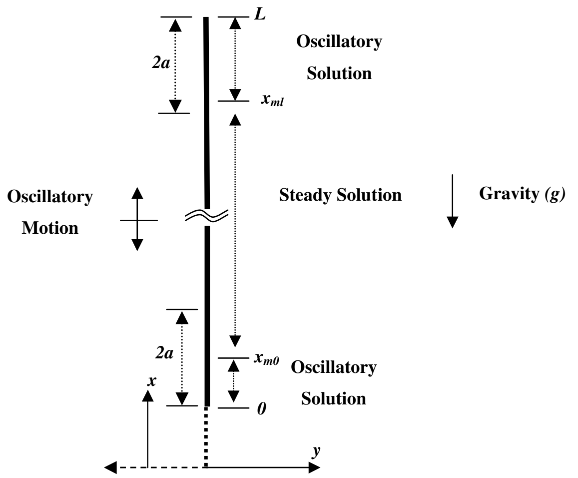

Fig. 1. Schematic of analytical model.

particle that past the leading edge first arrives at that point before the plate changes direction. This mechanism is periodic, and will contribute to the AC heat transfer component. In other words, for points on the surface that are never reached by fluid particles, or the fluid velocity there is too small, as is the case near shear reversal, the quasi-steady solution is replaced by diffusive solution in presence of gravity force. Based on the above assumptions, the continuity, momentum, and energy equations with their appropriate boundary conditions are given by:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=v\frac{\partial^2 u}{\partial y^2}+\frac{g(\rho(T_w)-\rho(T_\infty))}{\rho(c)}+\frac{du_0}{dt} \tag{2}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

where u_0 is the fluid velocity outside the boundary layer relative to the selected coordinate axis,

$$u_0=a\omega\cos\omega t \tag{4}$$

And the boundary conditions given by:

$$u(x,0,t)=v(x,0,t)=0 \tag{5a}$$

$$u(x,\infty,t)=u_0 \tag{5b}$$

$$T(x,0,t)=T_w \tag{5c}$$

$$T(x,\infty,t)=T_\infty \tag{5d}$$

Since boundary-layer linearization cannot be applied in cases involving shear reversal, a solution will be developed near the plate leading edge based on the fact that, the flow there is essentially Blasius, and the situation is that of determining the effect of the buoyancy forces on the upward or downward forced convection solution. On the other hand, and far away from the plate leading edge, the layer is mainly Stokes, and the situation will be of determining the rate of heat transfer across the layer in presence of buoyancy force.

2.3. Development of the pseudo-steady state solution (DC)

Away from the plate leading edge, an unsteady problem is formed with initial conditions that evolves to "pseudo-steady state solution" at large times. The intent is to develop an approximate expression for the steady state component without residing to the solution of the initial value problem. In doing so the steady state heat and momentum equations will be used with the attempt to evaluate the steady state effect of oscillations. Eqs. (2) and (3) reduce to:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=v\frac{\partial^2 u}{\partial y^2}+g\lambda\varphi \tag{6}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \tag{7}$$

where φ is the dimensionless temperature defined by: $\varphi=(T-T_\infty)/(T_w-T_\infty)$ and λ, the volumetric expansion coefficient:

$$\lambda=T_\infty/\rho(T_\infty)[\partial\rho/\partial T]_{T=T_\infty} \tag{8}$$

Introducing the dimensionless Pohlhausen variable,

$$\eta=[g\lambda/4v^2]^{1/4}y/x^{1/4} \tag{9}$$

and the stream function

$$\psi=4v[g\lambda/4v^2]^{1/4}x^{3/4}p(\eta) \tag{10}$$

where the function p is unknown. In terms of the new variable, the velocity components are,

$$u=4v[g\lambda/4v^2]^{1/2}p'(\eta) \tag{11}$$

$$v=v[g\lambda/4v^2]^{1/4}(\eta p'-3p)/x^{1/4} \tag{11a}$$

and Eqs. (6) and (7) take the form:

$$p'''+3pp''-2(p')^2+\varphi=0 \tag{12}$$

$$\varphi''+3(Pr)p\varphi'=0 \tag{13}$$

and, finally, for the boundary conditions of the problem, we obtain the expressions:

$$p=p'=0,\quad \varphi=1,\quad \text{for } \eta=0 \tag{14}$$

$$p'=p_0,\quad \varphi=0 \quad \text{for } \eta=\infty \tag{15}$$

For large values of Pr number, the thermal boundary layer becomes thin and consequently only the fluid layer next to the plate contributes significantly to the heat transfer resistance. Such assumption was found to be valid by Ruckentein and Rajagopalan [56] even for values of Pr number near the order of unity. Accordingly, solution of Eq. (13) takes the following form:

$$\varphi = 1 - \frac{\left[\int_0^\eta \exp\left\{-3Pr\int_0^\eta p\,\mathrm{d}\eta\right\}\mathrm{d}\eta\right]}{\left[\int_0^\infty \exp\left\{-3Pr\int_0^\eta p\,\mathrm{d}\eta\right\}\mathrm{d}\eta\right]} \tag{16}$$

In Eq. (16), the function p would be determined based on the fact that the integrals converge rapidly, and its magnitude is determined mainly by the value of p for small values of η. Therefore, and without introducing any significant error, the boundary conditions at infinity given in Eq. (15) may be replaced by new boundary conditions, which is fulfilled at a finite distance $\eta = \eta_0$ from the surface:

$$p' = u(\eta_0)/[4v[g\lambda/4v^2]^{1/2}] \tag{17a}$$

$$\varphi = 0, \quad \eta = \eta_0 \tag{17b}$$

And $u(\eta_0)$ and η_0 are given by [57],

$$u(\eta_0) = aw\{1 - \mathrm{e}_0^{-\eta}\cos(wt - \eta_0)\} \tag{17c}$$

$$\eta_0 = y_0(w/2v)^{1/2} \tag{17d}$$

Owing to the rapid convergence of the integrals in Eq. (16), we can write a series expansion in powers of η for the function p for $\eta < \eta_0$, and retain only the first terms of the expansion. In view of the boundary conditions in Eq. (14), the series expansion of p is:

$$p = (\beta/2)\eta^2 + (\gamma/6)\eta^3 + \cdots \tag{18}$$

Substituting the expression for p from Eq. (18) into the solution given in Eq. (16), and using only one term as first approximation:

$$\varphi = 1 - \frac{\left[\int_0^\eta \exp\left\{-\beta Pr\frac{\eta^3}{2}\right\}\mathrm{d}\eta\right]}{\left[\int_0^\infty \exp\left\{-\beta Pr\frac{\eta^3}{2}\right\}\mathrm{d}\eta\right]} \tag{19}$$

Which yields the following solution:

$$\varphi \approx 1 - [(\beta Pr/2)^{1/3}\eta]\Gamma(4/3) \tag{20}$$

Using the boundary conditions in (17b):

$$\varphi = 0 \approx 1 - [(\beta Pr/2)^{1/3}\eta_0]\Gamma(4/3) \tag{21}$$

Substituting the temperature distribution given by Eq. (20) into Eq. (12):

$$p''' + 3pp'' - 2(p')^2 + 1 - [(\beta Pr/2)^{1/3}\eta]\Gamma(4/3) = 0 \tag{22}$$

Using the method of successive approximation, and restricting the solution to the lower powers of η, an approximate solution of (22), may be written in the form:

$$p = \beta\eta^2/2 - \eta^3/6 + [(\beta Pr/2)^{1/3}\eta^4]/[(24)\Gamma(4/3)] \tag{23}$$

From the boundary condition in (17a),

$$\beta\eta_0 - \eta_0^2/2 + [(\beta Pr/2)^{1/3}\eta_0^3]/[(6)\Gamma(4/3)] - u(\eta_0)/[4v[g\lambda/4v^2]^{1/2}] = 0 \tag{24}$$

Solving (24) with (21), and reverting to dimensional variables, the temperature distribution can be determined using Eq. (20) in the form:

$$\Delta T = \Delta T_\mathrm{w}(\beta Pr/2)^{1/3}[g\lambda/4v^2]^{1/4}y/x^{1/4} \tag{25}$$

2.4. Solution near the leading edge (oscillatory—AC component)

The oscillatory solution will be determined, for simplicity, for one cycle oscillation, where the plate reverse its direction at time $t = 0$. For a point x on the surface, and long before reversal, fluid velocity, and consequently the convection term, are large in comparison with unsteady diffusion, and the temperature distribution can be approximated by a "quasi-steady" solution. This condition will continue until a time $-t_1(x)$ when fluid particles which had passed the leading edge at $x = 0$ first failed to arrive at x before flow reversal. For $t > -t_1(x)$, the fluid velocity is too small to an extend where convection becomes relatively unimportant, and the temperature distribution can be approximated by a "quasi-diffusive" solution. This condition will continue through shear reversal until time $t_2(x)$ when the fluid velocity becomes sufficiently large again, and a quasi-steady layer growing from the opposite edge at $x = L$ arrives at point x, and a quasi-steady solution is applied again. If conditions are such that no fluid particles arrives to x from L, then the quasi-diffusive solution will continue through the second reversal until a quasi-steady layer reaches x from $x = 0$. For that part of the surface where the effect of neither edge is felt, the temperature distribution will be given by the steady solution in Eq. (25). Applying the above assumptions:

(a) *For times $t < -t_1(x)$, and $t > t_2(x)$*: which represents times long before and after shear reversal, Eq. (3), can be solved by omitting the unsteady term, and approximating the velocity by the sum of its oscillatory and natural convection components. Introducing the similarity variable ξ:

$$\xi = y\left[S^{1/2}\Big/\left(9\alpha\int_0^x S^{1/2}\,\mathrm{d}x\right)\right]^{1/3} \tag{26}$$

Eq. (3) can be written as:

$$\varphi'' + 3\xi^2\varphi' = 0 \tag{27}$$

where,

$$u = yS, \quad v = -(1/2)y^2S' \tag{28}$$

$$S = S_\mathrm{f} + S_\mathrm{n} = (\partial u/\partial y)_{y=0} \tag{29}$$

$$S_\mathrm{f} = 0.332 u_0^{3/2}/(x\nu)^{1/2} \tag{30}$$

$$S_\mathrm{n} = 4\beta\nu[g\lambda/4\nu^2]^{3/4} x^{1/4} \tag{31}$$

and the resulting solution leads to:

$$\Delta T = \Delta T_\mathrm{w}\left\{1 - [1/\Gamma(4/3)]\int_0^{\xi} \mathrm{e}^{-\xi^3}\,\mathrm{d}\xi\right\} \tag{32}$$

(b) *For times $-t_1(x) < t < t_2(x)$*: which represent the period near shear reversal, the convective term in Eq. (3) becomes small and the temperature distribution is given by the unsteady diffusion,

$$\frac{\partial T}{\partial t} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{33}$$

The appropriate solution of (33) is:

$$\Delta T = \Delta T_\mathrm{w}\mathrm{erfc}\{y[4\alpha(t(x) + t_i(x))]^{-1/2}\} \tag{34}$$

where $t_i(x)$ is the initial conditions required to solve (33), and represents the time when pure diffusion has started or the "virtual origin of the diffusion". Eqs. (32) and (34) can be used for determining the oscillatory heat transfer component by determining the transition times, $-t_1(x)$, $t_i(x)$, and $t_2(x)$ together with maximum penetration distance, or the leading edge effect. The latter is determined from:

$$x_{m0} = y\int_{-\pi/\omega}^{0} S\,\mathrm{d}t \quad \text{for } -\pi/\omega < t < 0 \tag{35}$$

and

$$x_{ml} = y\int_{0}^{\pi/\omega} S\,\mathrm{d}t \quad \text{for } 0 < t < \pi/\omega \tag{36}$$

Since u is sheared proportional to y and does not take a unique value, the criteria proposed by Pedley [38] for selecting the value of y at the centre of mass will be applied. The centre of mass $\breve{y}$ is defined by:

$$\breve{y} = \int_0^\infty yT\,\mathrm{d}y \Big/ \int_0^\infty T\,\mathrm{d}y \tag{37}$$

Using Eq. (32), $\breve{y}$ can be determined from,

$$\breve{y} = 0.37[9\alpha x/S(t)]^{1/3} \tag{38}$$

The time parameters $-t_1(x)$ and $t_2(x)$ are determined by solving Eqs. (35) and (36) for each point on the surface, x_T, where there is a transition between quasi-steady and diffusive solution in the range $0 < x < x_{m0}$, and $(L - x_{ml}) < x < x_{ml}$. The virtual origin of diffusion $t_i(x)$, is determined by the requirement that the centre of mass of the temperature distribution be continuous at the transition time. From Eqs. (32) and (34) this continuity requires that,

$$(\pi/4)(t_0(x) - t_1(x)) = [9\alpha x/S(-t)_1(x)]^{2/3}\tilde{\xi}'^2] \tag{39}$$

The overall time dependent heat transfer coefficient can then be determined from:

$$h(t) = (\alpha/\Delta T_\mathrm{w} L)\int_0^l [-\partial T/\partial y]_{y=0}\,\mathrm{d}x \tag{40}$$

where the integrand is given by the solutions in Eqs. (25), (32) and (34) to be:

$$[-\partial T/\partial y]_{y=0} = \Delta T_\mathrm{w}[-S(t)/9\alpha x]^{1/3}/\Gamma(4/3)$$
$$\text{for } t \leqslant -t_1 \tag{41a}$$
$$= \Delta T_\mathrm{w}[-S(t)/9\alpha(1-x)]^{1/3}/\Gamma(4/3)$$
$$\text{for } t \geqslant t_2 \tag{41b}$$
$$= \Delta T_\mathrm{w}[\pi\alpha(t_0(x) + t_1(x))]^{-1/2}$$
$$\text{for } -t_1 < t < t_2 \tag{41c}$$
$$= \Delta T_\mathrm{w}(\beta Pr/2)^{1/3}[g\lambda/4\nu^2]^{1/4}/x^{1/4}$$
$$\text{for all } t, x_{m0} < x < x_{ml} \tag{41d}$$

$h(t)$ is evaluated by integrating either (41a) from 0 to x_T and (41c) from x_T to L, if $S(t) > 0$, or (41c) from 0 to x_T and (41b) from x_T to L, if $S(t) < 0$. If conditions are such that there exist a section on the surface $x_{m0} < x < x_{ml}$ where the effect of neither leading edges is felt, then Eq. (41d) will be integrated from x_{m0} to x_{ml}, for the entire cycle to determine the steady state component. The oscillatory component in this case is evaluated by integrating either (41a) from 0 to x_T and (41c) from x_T to x_{m0}, and from x_{ml} to L if $S(t)>0$, or (41c) from 0 to x_{m0} and from x_{ml} to x_T and (41b) from x_T to L, if $S(t) < 0$. The time average heat transfer coefficient is then determined from:

$$h_\mathrm{v} = (1/2\pi)\int_{-\pi/\omega}^{\pi/\omega} h(t)\,\mathrm{d}t \tag{42}$$

3. Results and discussion

3.1. Time average heat transfer

The rate of heat transfer at the surface of stationary vertical plate is controlled by the natural convection boundary layer created by the density difference due to temperature change near its surface. When an oscillatory motion is created between the plate and the adjacent fluid, the time average vibrational heat transfer coefficient (h_v), increases with increasing either the frequency or amplitude of vibration. Fig. 2 shows the effect of oscillation on the time average heat transfer rate expressed in terms of the enhancement factor E defined by,

$$E = Nu_\mathrm{v}/Nu_\mathrm{n} \tag{43}$$

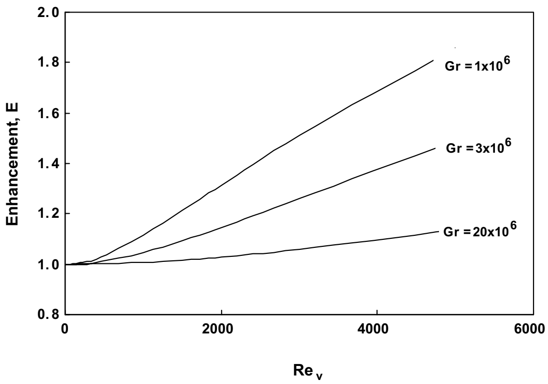

Fig. 2. Effect of vertical oscillations on heat transfer enhancement (E).

where Nu_v, and Nu_n represents Nusselt numbers under oscillating and stationary conditions respectively. As seen from Fig. 2 oscillations increases E with the extent of enhancement being most pronounced for small values of Grashof number Gr. The influence of oscillations diminishes as Gr increases, where free convection effects dominate over the oscillation effects, and the opposite situation prevails at low values of Gr. Referring to Fig. 3a and b further substantiates this argument, where the effect of oscillation is considerably reduced as Gr increases and all the curves tend to merge with the pure convection curve (Fig. 3a). Similarly, for low values of Gr, the curves tend to merge with the pure forced convection curve (Fig. 3b). In this region however, free convection effects are not completely masked by the oscillations, and both free and forced convection effects influence the heat transfer rate.

The predicted increase in heat transfer rate due to oscillation agrees well the experimental measurements of Prasad and Ramanathan [45], and Eshgy et al. [49], for heat transfer from vertically oscillating flat surfaces as shown in Figs. 4 and 5. The increase, as explained earlier, is attributed to the larger temperature gradient across the thin Stokes layer formed by the surface oscillatory motion, in combination of the enhanced transfer at the leading edges of the plate, which also increases with both amplitude and frequency of oscillation. The model predictions converges to the pure natural convection solution as $aw \to 0$, and to the pure oscillatory forced convection in absence of buoyancy force. In the first case, $p' \to 0$, in Eq. (17a) and the average heat transfer coefficient is calculated by integrating Eq. (41d) over the entire surface, which for $Pr = 0.72$ leads to the standard natural convection solution [57],

$$Nu_n = 0.516(Gr \cdot Pr)^{0.25} \tag{44}$$

In the second case, Eq. (41d) $\to 0$, and the time average heat transfer coefficient is calculated by integrating either Eq. (41a) or (41b) over the entire surface, which in absence of natural convection give the same time average value for both halves of the oscillatory cycle. Using the time average oscillatory velocity ($u_{ave} = 0.64a\omega$), and for $Pr = 0.72$, the integration result in the standard equation for heat transfer over a flat plate given by [57],

$$Nu_f = 0.632Re^{0.5}Sc^{0.333} \tag{45}$$

Eq. (45) can also be written in terms of the vibrational Reynolds number $Re_v = a\omega L/\nu$ as,

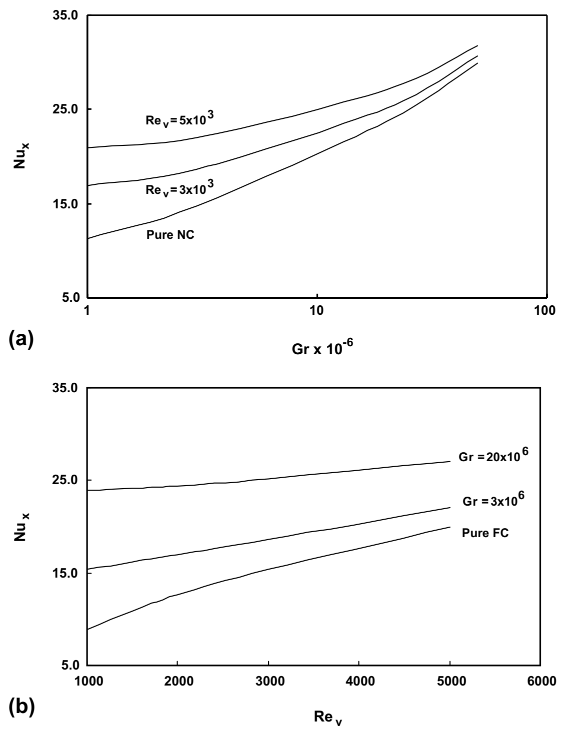

Fig. 3. (a) Effect of Gr on heat transfer rate in presence of oscillatory motion and (b) effect of Re_v on heat transfer rate.

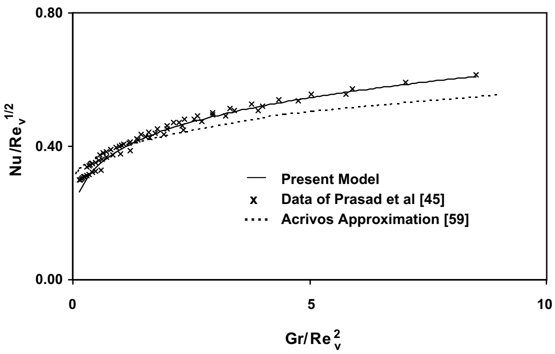

Fig. 4. Comparison of models ($Pr = 0.72$).

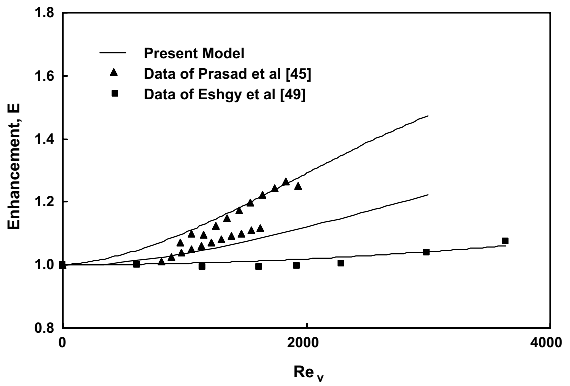

Fig. 5. Effect of oscillatory motion on heat transfer enhancement at vertical surfaces: comparison of model with experimental data.

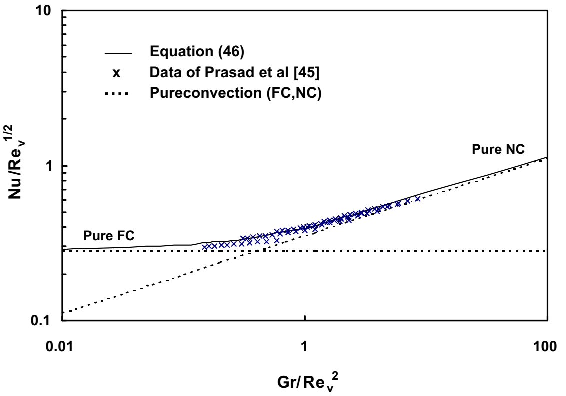

Fig. 6. Combined natural and oscillatory forced convection at vertical surfaces ($Pr = 0.72$).

$$Nu_{\rm f} = 0.506 Re_{\rm v}^{0.5} Sc^{0.333} \tag{45a}$$

The above suggests that, it is possible from a practical point to express the effect of oscillation on heat transfer at vertical surface in terms of mixed natural and forced oscillatory convections, and to estimate $Nu_{\rm v}$ using a mixed convection correlation given by [58],

$$Nu_{\rm v} = [Nu_{\rm n}^3 + Nu_{\rm f}^3]^{1/3} \tag{46}$$

Fig. 6 shows a plot of Eq. (46) in terms the similarity parameter $Gr/Re_{\rm v}^2$ used by Acrivos [59] to indicate the relative influence of forced and natural convections in a mixed convection process. This influence becomes significant when the Reynolds number is of the same order of magnitude as the Grashof number. Furthermore, it is expected that at any value of $Gr/Re_{\rm v}^2$ between the limits of pure oscillatory and pure free convection, the Nusselt number Nu would be higher than it would be in either of these modes alone. The agreement between Eq. (46) and the experimental data shown in Fig. 6 further supports the argument of using mixed convection correlation for predicting the effect of oscillation on the time average heat transfer rate at vertical surfaces. Under such conditions, and as shown in Fig. 6, the free convection limit is approached at approximately $Gr/Re_{\rm v}^2 = 3$, while the oscillatory forced convection limit is approached at approximately $Gr/Re_{\rm v}^2 = 0.1$. This suggests that oscillations enhance heat transfer rate at vertical surfaces in the region of $0 < Gr/Re_{\rm v}^2 < 3.0$, and that the influence of natural convection on heat transfer enhancement becomes significant for values of $Gr/Re_{\rm v}^2 > 0.1$.

3.2. Oscillatory heat transfer

Analysis of the transient heat transfer data revealed several observations that could not be explained by Leveque quasi-steady analysis. First, and as shown in Figs. 7 and 8, the oscillatory (AC) component, is relatively small in comparison to the *pseudo-steady* (DC) component, and never reaches zero as predicted by the quasi-steady analysis. Furthermore, the ratio of the AC to DC component was also noticed to decrease with increasing L/a, as shown in Fig. 9, where the AC component is ~45% of the total value for $L/a = 0.25$ in Fig. 7 compared to ~24% for $L/a = 1.5$ in Fig. 8. This observation could be explained using the present analysis to be due to the presence of a quasi-diffusive mechanism that accounts for the relatively large DC component that does not reach zero at reversal time. Meanwhile, the dependency of the AC to DC ratio on L/a can be attributed to fact that the former is determined by the fraction of surface being reached by fluid particles from the leading at the beginning of each half cycle, which varies with the amplitude of oscillation.

The second observation is the complex harmonic structure of the oscillatory component, which behaved as a composite signal of two sub-harmonics with characteristic frequencies f and $2f$ that varied in amplitude

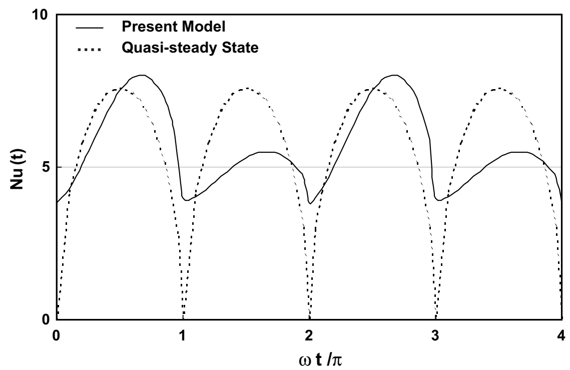

Fig. 7. Oscillatory heat transfer ($f = 5$ Hz, $L/a = 0.25$) comparison with quasi-steady state analysis.

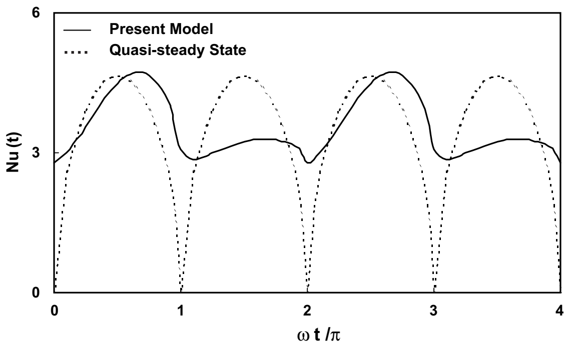

Fig. 8. Oscillatory heat transfer (f = 5 Hz, L/a = 1.5) comparison with quasi-steady state analysis.

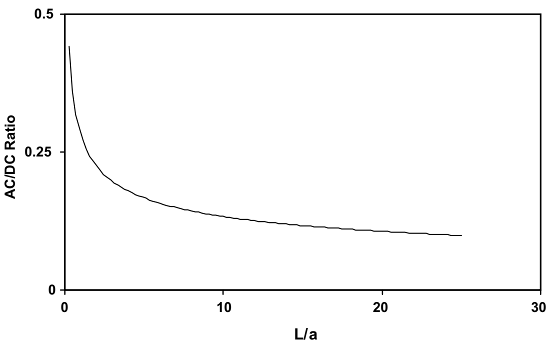

Fig. 9. Effect of L/a on the oscillatory heat transfer component.

depending on the surface height and the oscillation parameters. For example, while the amplitude of the second harmonic in Fig. 7 is ~40% of the first, it decreases as L/a increases to ~26% in Fig. 8. This can be attributed to the opposite effect of the buoyancy force in each half of the oscillating cycle, which increases with Gr until a situation occurs where the steady effect becomes much higher than the oscillatory term to mask the second harmonic during the "opposing-flow" half cycle. Such behavior is different from that predicted by the quasi-steady solution as shown in Figs. 7 and 8.

Although no experimental data for transient heat transfer at vertical surfaces in presence of oscillatory motion is available for comparison, the predicted temporal heat transfer agrees, qualitatively, with the data of Gomaa et al. [15], and Liu et al. [44] for mass transfer at vertically oscillating flat surfaces, which showed a definite oscillatory component (AC) superimposed on a much larger steady one (DC) throughout the vibratory cycle, that did not follow the oscillatory velocity, which periodically approaches zero at reversal times. This is further confirmed by the interferograms of the heat transfer boundary layer at a longitudinally vibrating vertical plate obtained by Prasad and Ramanathan [45], which show an almost steady thickness boundary layer that did not change significantly throughout the vibratory cycle. Therefore, and based on the relatively good agreement with the time average experimental data, it can be assumed that the present model adequately describe the heat transfer and fluid mechanics at vertical surfaces in presence of oscillatory motion. The accuracy of the model however, is expected to be highest at larger L/a ratios due in part to the weaker effect of the leading edge at small amplitude to surface ratios, which would decrease the associated error resulting from the "abrupt take-over" assumption between the quasi-steady and diffusive mechanisms near reversal times. Another factor would also be the weaker effect of the error introduced by the "so-called wake", where a relatively low concentration fluid is carried back over the surface during reversal time, a factor was not addressed in the present analysis, and could lead to lager error for large a/L ratios.

4. Conclusions

Oscillatory motion enhances the rate of heat transfer at vertical surfaces with the increase being highest at higher frequencies and amplitude of oscillation. The enhancement can be attributed to two heat transfer mechanisms. First, and away from the surface edges, heat transfer occurs primarily by diffusion across a layer which is much thinner than natural convection layer in absence of oscillation (Stokes layer). Second, and near the surface edges, a quasi-steady mechanism is dominant and contributes to the formation of an AC component with characteristic frequency that varies in amplitude and frequency from f for large values of L/a to $2f$, with increasing the amplitude to height ratio. From a practical point of view, it is possible to calculate the time average heat transfer coefficient using a mixed convection expression with the forced convection term calculated based on the time average oscillatory velocity. Under such conditions, the effect of oscillations on heat transfer enhancement is most pronounced in the region of $0 < Gr/Re_v^2 < 3.0$, beyond which heat transfer becomes mainly determined by natural convection. On the other hand, for values of $Gr/Re_v^2 < 0.1$, the effect natural convection becomes insignificant, and that heat transfer rate is determined mainly by forced oscillatory convection. In the region $0.1 < Gr/Re_v^2 < 3.0$, the influence of both natural and oscillatory forced convections must be taken into account for determining the time average heat transfer rate at vertical surfaces in presence of oscillatory motion. The fact that modest heat transfer enhancement is observed under the present conditions (<2) is due to the small convective term in the energy equation, which as shown by Kurzweg [4–6] that much larger heat transfer enhancement can be achieved by increasing the axial temperature gradient in presence of oscillatory motion.

References

[1] X. Ni, M.R. Mackley, A.P. Harvey, P. Stonestreet, M.H.I. Baird, N.V. Rama Rao, Mixing through oscillations and pulsations—a guide to achieve process enhancements in the chemical and process industries, Trans. Inst. Chem. Eng. 81 (2003) 373–383.

[2] T. Zhao, P. Cheng, A numerical solution of laminar forced convection in a heated pipe subjected to a reciprocating flow, Int. J. Heat Mass Transfer 38 (16) (1995) 3011–3022.

[3] W.L. Cooper, K.T. Yang, V.W. Nee, Fluid mechanics of oscillatory and modulated flows and associated applications in heat and mass transfer—a review, J. Energy Mass Transfer 15 (1993) 1–19.

[4] U.H. Kersweg, Enhanced heat conduction in oscillatory flow within parallel channels, J. Fluid Mech. 156 (1985) 291–300.

[5] U.H. Kersweg, Enhanced heat conduction in fluid subjected to sinusoidal oscillations, J. Heat Transfer (ASME) 107 (1985) 459–462.

[6] U.H. Kersweg, Temporal and spatial distribution of hear flux in oscillating flow subject to an axial temperature gradient, Int. J. Heat Mass Transfer 29 (12) (1986) 1969–1977.

[7] M. Ozawa, M. Shinoki, K. Nagoshi, E. Serizawa, Scaling of heat transfer characteristics in an oscillating flow, J. Enhanced Heat Transfer 10 (3) (2003) 275–285.

[8] T. Zhao, P. Cheng, Heat transfer in oscillatory flowsAnnual Review of Heat Transfer, IX, 1998 (Chapter 7).

[9] G.C. Stephens, M.R. Mackley, Heat transfer performance for Bach oscillatory flow mixing, Experimental Thermal and Fluid Science 25 (2002) 583–594.

[10] M. Kaviany, Performance of a heat exchanger based on enhanced heat diffusion in fluids by oscillation: analysis, J. Heat Transfer 112 (1) (1990) 49–55.

[11] M. Kaviany, M. Reckker, Performance of a heat exchanger based on enhanced heat diffusion in fluids by oscillation: experiment, J. Heat Transfer 112 (1) (1990) 56–63.

[12] J.E. Dec, J.O. Keller, V.S. Arpaci, Heat transfer enhancement in the oscillating turbulent flow of a pulse combustor tail pipe, Int. J. Heat Mass Transfer 35 (9) (1992) 2311–2325.

[13] N.G. Carpenter, E.P.L. Roberts, Mass transport and residence time characteristics of an oscillatory flow electrochemical reactor, Trans. I. Chem. E. 77 (A) (1999) 212–217.

[14] N. Siamak, B.J. Bellhouse, Effect of oscillatory flow on the performance of a novel cross flow affinity membrane device, Biotechnol. Prog. 13 (1997) 113–116.

[15] H.G. Gomaa, A.M. Al Taweel, J. Landau, Mass transfer enhancement at vibrating electrodes, Chem. Eng. J. 97 (2004) 141–149.

[16] L. Bronfenbrener, L. Grinis, E. Korin, Experimental study of heat transfer intensification under vibration condition, Chem. Eng. Technol. 24 (2001) 367–371.

[17] A.M. Thomas, R. Narayanan, A comparison between mass transfer in boundary and pressure driven oscillatory flow, Int. J. Heat Mass Transfer 45 (2002) 4057–4062.

[18] C.K. Drummond, F.A. Lyman, Mass transfer from a sphere in an oscillating flow with zero mean velocity, Comput. Mech. 6 (1990) 315–326.

[19] A.M. Al Taweel, J. Landau, Effect of oscillatory motion on mass transfer between a solid sphere and fluids, Can. J. Chem. Eng. 54 (1976) 532–540.

[20] M.E. Ralph, Oscillatory flow in wavy-walled tubes, J. Fluid Mech. 168 (1986) 515.

[21] M. Mackley, Use of oscillatory flow to improve performance, Chem. Eng. (1986) 18–43.

[22] V.G. Rodgers, R.E. Sparks, Reduction of membrane fouling in the ultrafiltration of binary protein mixtures, AIChE J. 37 (1991) 39–55.

[23] K. Abel, Influence of oscillatory flows on protein ultrafiltration, J. Mem. Sci. 133 (1997) 39–55.

[24] T. Nishimura, N. Oka, Y. Yoshinak, K. Kunitsugu, Influence of imposed oscillatory frequency on mass transfer enhancement of grooved channels for pulsatile flow, J. Heat Mass Transfer 43 (2000) 2365–2374.

[25] M. Ozawa, T. Takifuji, A. Kawamoto, Convective heat transfer induced by fluid oscillations, Heat Transfer—Jpn. Res. 26 (1) (1997) 39–53.

[26] S. Nishio, X.H. Shi, W.M. Zhang, Oscillation-induced heat transport characteristics along liquid-columns of oscillation-controlled heat transport tubes, Int. J. Heat Mass Transfer 38 (13) (1995) 2457–2470.

[27] U.H. Kurzweg, J. Chen, Heat transport along oscillating flat plate, J. Heat Transfer 110 (1988) 789–790.

[28] O. Aaboubi, I. Citti, J. Chopart, C. Gabrielli, A. Olivier, B. Tribollet, Thermoelectrical transfer function under thermal laminar free convection at vertical electrode, J. Electrochem. Soc. 147 (10) (2000) 3808–3818.

[29] S. Khandekar, P. Charoensawan, M. Manfred, P. Terdtoon, Closed loop pulsating heat pipes—part B: visualization and semi-empirical modeling, Appl. Thermal Eng. 23 (16) (2003) 2021–2033.

[30] Z. Jon Zuo, M.T. North, K.L. Wert, High heat flux heat pipe mechanism for cooling of electronics, IEEE Trans. Components Packag. Technol. 24 (2) (2001).

[31] M. Shafii, A. Faghri, Y. Zhang, Thermal modeling of unlooped and looped pulsating heat pipes, J. Heat Transfer ASME 123 (2001) 1159–1172.

[32] S.B. Nikol'skaya, Y.S. Chumakov, Experimental investigation of pulsation motion in a free-convection boundary layer, High Temp. 38 (2) (2000) 231–237.

[33] M.A. Leveque, Les Lois de la Transmission de Chaleur par Convection, Annis Mines 13 (1928) 201–299.

[34] M.J. Lighthill, The response of laminar skin friction and heat transfer to fluctuations in the stream velocity, Proc. R. Soc. 224A (1954) 1–23.

[35] G. Fortuna, T.J. Hanratty, Frequency response of the boundary layer on wall transfer probes, Int. J. Heat Mass Transfer 14 (1971) 499–1507.

[36] L. Talbot, J.L. Steinert, The frequency response of electrochemical wall shear probes in pulsatile flow, Trans. ASME 109 (1987) 60–64.

[37] C. Deslouis, O. Gil, B. Tribollet, Frequency response of electrochemical sensors to hydrodynamic fluctuations, J. Fluid Mech. 25 (1990) 85–100.
[38] T.J. Pedley, Heat transfer from a hot film in reversing shear flow, J. Fluid Mech. 78 (3) (1976) 513–534.
[39] C.B. Watkins, I.H. Herron, Laminar shear layer due to a thin flat plate oscillating with zero mean velocity, Trans. ASME J. Fluids Eng. 100 (1976) 367–373.
[40] P. Kaiping, Unsteady forced connective heat transfer from a hot film in non-reversing and reversing shear flow, Int. J. Heat Mass Transfer 26 (1983) 545–557.
[41] A.A. Van Steenhoven, F.J.H.M. Van Beucken, Dynamical analysis of wall shear rate measurements, J. Fluid Mech. 231 (1991) 599–614.
[42] Z. Mao, J. Hanratty, Analysis of wall shear probes in large amplitude unsteady flows, Int. J. Heat Mass Transfer 34 (1) (1991) 281–290.
[43] Z. Mao, J. Hanratty, Influence of large amplitude oscillations on turbulent drag, AIChE J. 40 (10) (1994) 1601–1610.
[44] M.B. Liu, E.M. Rudnick, G.M. Cook, N.P. Yao, Mass transfer at longitudinally vibrating vertical electrodes, J. Electrochem. Soc. 129 (1982) 1955–1959.
[45] K.K. Prasad, V. Ramanathan, Heat transfer by free convection from a longitudinally vibrating vertical plate, Int. J. Heat Mass Transfer 15 (1972) 1213–1223.
[46] C.V. Rama Raju, A. Ramalinga Sastry, G.J.V.J. Raju, Ionic mass transfer at vibrating plates, Indian J. Technol. 7 (1969) 35–38.
[47] W.K. Chan, S.L. Lee, C.Y. Liu, Effects of frequency and amplitude of oscillation on low Reynolds number pulsating flow in a circular pipe, Eng. Comput. 19 (1–2) (2002) 119–136.
[48] D.W. Wundrow, M.E. Goldstein, Effect on laminar boundary layer of small amplitude streamwise vorticity in the upstream flow, J. Fluid Mech. 426 (2001) 229–263.
[49] S. Eshgy, V.S. Arpaci, J.A. Clark, The effect of longitudinal oscillations on free convection from vertical surfaces, J. Appl. Mech. 183–191 (March) (1965).
[50] N. Riley, E.H. Trinh, Steady streaming in an oscillatory inviscid flow, Phys. Fluids 13 (7) (2001) 1956–1961.
[51] R.S. Alassar, H.M. Badr, H.A. Mavromatis, Heat convection from a sphere placed in an oscillating free stream, Int. J. Heat Mass Transfer 42 (7) (1999) 1289–1305.
[52] N.V. Vighnesam, S.N. Ray, V.M. Soundalgekar, Oscillating plate temperature effects on mixed convection flow past a semi-infinite vertical porous plate, Def. Sci. J. 51 (4) (2001) 415–419.
[53] S. Revenkar, Free convection effect on the flow past an impulsively started or oscillating infinite vertical plate, Mech. Res. Comms. 27 (2) (2000) 241–246.
[54] W.S. Fu, W.J. Shieh, Transient thermal convection in an enclosure induced simultaneously by gravity and vibration, Int. J. Heat Mass Transfer 36 (2) (1993) 437–452.
[55] H. Kimoto, H. Ishida, Vibration effects on the average heat transfer characteristics of the natural convection field in a square enclosure, Heat Transfer—Asian Res. 29 (7) (2000) 545–558.
[56] E. Ruckenstein, R. Rajagopalan, A simple alegebraic method for obtaining the heat or mass transfer coefficients under mixed convection, Chem. Eng. Comm. 4 (1980) 15–39.
[57] H. Schlichiting, Boundary-Layer Theory, seventh ed., McGraw-Hill, New York, 1979, p. 93.
[58] E. Ruckenstein, Prediction of rates of heat and mass transfer in complex situations by interpolating between simpler limiting cases, Chem. Eng. Sci. 37 (10) (1982) 1505–1511.
[59] A. Acrivos, Combined laminar free- and forced-convection heat transfer in external flows, AIChE J. 4 (1958) 285–289.